

# Neutrino mixing and large $CP$ violation in $B$ physics

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We show that in seesaw models of neutrino mass in a SUSY  $SO(10)$  context, the observed large mixing in atmospheric neutrinos naturally leads to large  $b$ - $s$  transitions. If the associated new  $CP$  phase turns out to be large, this SUSY contribution can drastically affect the  $CP$  violation in some of the  $B$  decay channels yielding the  $\beta$  and  $\gamma$  angles of the unitarity triangle. They can even produce sizable  $CP$  asymmetries in some decay modes which are not  $CP$  violating in the standard model context. Hence the observed large neutrino mixing makes observations of the low energy SUSY effect in some  $CP$  violating decay channels potentially promising in spite of the agreement between the standard model and data in  $K$  and  $B$  physics so far.

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## I. INTRODUCTION

In the past couple of years we have obtained three major pieces of information on flavor physics from experiment: large neutrino mixing in atmospheric neutrinos, the existence of "direct  $CP$  violation" in the neutral kaon system  $\epsilon'/\epsilon$  and observation of  $CP$  violation in  $B \rightarrow \psi K_s$ . Only the first of these three results clearly calls for new physics beyond the standard model (SM) for its explanation. As for  $\epsilon'/\epsilon$ , the theoretical uncertainties surrounding the SM predictions prevent any firm conclusion. Nevertheless, it is reassuring to notice that, in the most familiar and promising extension of the SM, the minimal supersymmetric SM (MSSM), there is indeed room for a large contribution to  $\epsilon'/\epsilon$  even in the presence of a tiny deviation from flavor universality in the terms which break supersymmetry (SUSY) softly [1]. As for the observed  $CP$  violation in  $B$  physics, the results are in agreement with the SM expectation, but they leave open the possibility of large  $CP$  violating contributions from new physics (in particular the MSSM) in other decay channels which should be observable in the near future at  $B$  factories or hadron colliders.

The new physics involved in explaining the atmospheric neutrino observations must provide a mass to neutrinos and guarantee at least one maximal mixing in the leptonic sector. One of the best candidates to accomplish these tasks is the seesaw mechanism [2,3]. It has been known for a long time that, at variance with what occurs in the non-SUSY case, the SUSY version of the seesaw mechanism can potentially lead to large lepton flavor violating (LFV) effects [4].<sup>1</sup> Obviously, then, if one combines the SUSY seesaw with the idea of some hadron-lepton unification, one may suspect that the

large mixing between second and third generation in the neutrino case entails not only some large LFV, but also some large mixing among quarks of the second and third generation which sit together with the leptons in a GUT multiplet. This is indeed what may happen for the right-handed quark supermultiplets in a SUSY  $SU(5)$  construction with a seesaw mechanism [6].

Motivated by the three above experimental observations and the above-mentioned theoretical considerations in a SUSY context, this paper addresses the following question: in a SUSY seesaw context where neutrino and up-quark couplings are the result of the grand unified theory (GUT), how large can the SUSY contributions to  $CP$  violation in  $B$  physics be? So far it has been observed that, considering a model-independent parametrization of the  $CP$  violating SUSY contributions in  $B$  physics, it is possible, compatibly with all the existing phenomenological constraints, to obtain sizeable effects. However, given that we have now a precise indication of large LFV effects in the neutrino sector, we find it timely and important to link this experimental fact to predictions for  $B$  physics in a motivated SUSY GUT context which accounts for the atmospheric neutrino results.

We find that (i) differently from  $SU(5)$ -like schemes where one has to assume the largeness of some neutrino coupling to infer large quark flavor violation (FV) from the observed large neutrino mixing, in an  $SO(10)$ -like context the link between this latter phenomenon and large  $b$ - $s$  transitions is automatically ensured, (ii) the mixing  $B_s$ - $\bar{B}_s$  can receive large SUSY contributions comparable, if not larger, than the SM contribution, (iii) some of the  $CP$  violating  $B$  decays which yield the  $\beta$  and  $\gamma$  angles of the unitarity triangle are strongly affected by the presence of the SUSY  $CP$  violating contributions, whilst other decays which in the SM are predicted to yield the same angles are essentially un-

<sup>1</sup>For recent works on LFV in SUSY seesaw models, see Ref. [5].

touched by SUSY, and (iv) there exist some decay channels which do not present any sizeable  $CP$  asymmetry in the pure SM which develop significant (and hopefully observable)  $CP$  signals thanks to the large SUSY contributions. The paper shows that, differently from some pessimistic common lore after the  $CP$  and flavor changing neutral current (FCNC) results in  $K$  and  $B$  physics so far, the important experimental finding in atmospheric neutrinos yields expectations of sizable deviations from the SM in some  $CP$  violating  $B$  decays in SUSY GUT schemes where neutrino masses arise from a seesaw mechanism.

## II. MAIN POINT

The main point of this paper is very simple. Consider an  $SO(10)$  grand-unified theory, which breaks to  $SU(5)$  at, e.g.,  $10^{17}$  GeV. The Yukawa coupling of “third-generation” neutrino is unified with the large top Yukawa coupling thanks to the  $SO(10)$  unification. However, the large mixing angle in atmospheric neutrino oscillation suggests that this “third-generation” neutrino is actually a near-maximal mixture of  $\nu_\mu$  and  $\nu_\tau$ . On the other hand, the “third-generation” charged leptons and down-type quarks have a relatively large unified bottom and tau Yukawa coupling, which is diagonal in the  $SU(5)$  multiplet that contains  $\nu_\tau$  by definition. Therefore, the  $SU(5)$  multiplet with the large top Yukawa coupling contains approximately

$$5_3^* = 5_\tau^* \cos \theta + 5_\mu^* \sin \theta, \quad (1)$$

where  $\theta \simeq 45^\circ$  is the atmospheric neutrino mixing angle, and

$$5_\tau^* = (b^c, b^c, b^c, \nu_\tau, \tau), \quad (2)$$

$$5_\mu^* = (s^c, s^c, s^c, \nu_\mu, \mu). \quad (3)$$

It is interesting that even a large mixing in right-handed down quarks does not appear in Cabibbo-Kobayashi-Maskawa (CKM) matrix simply because there is no charged-current weak interaction on right-handed quarks.

The top Yukawa coupling then generates an  $O(1)$  radiative correction to the mass of  $\tilde{s} \sin \theta + \tilde{b} \cos \theta$ , which leads to a large mixing between  $\tilde{s}$  and  $\tilde{b}$  at low energies. This large mixing in turn generates interesting effects in  $B$  physics. Examples include: large  $B_s$  mixing and  $CP$  violation, different “ $\sin 2\beta$ ” in  $B_d \rightarrow \phi K_s$  from that in  $B_d \rightarrow J/\psi K_s$  due to the  $CP$ -violating penguin operator, and different “ $\gamma$ ” values from different processes.

The rest of the paper is devoted to more details of this simple point. In particular, we demonstrate in the next section that one can write down semiconcrete models of  $SO(10)$  unification which lead to large  $\tilde{b}$ - $\tilde{s}$  mixing. It is important that such models do not necessarily cause too-large  $\mu \rightarrow e \gamma$  or other dangerous effects. Then we will discuss detailed consequences of large  $\tilde{b}$ - $\tilde{s}$  in  $B$  physics.

## III. $SO(10)$

### A. Framework

We are motivated by  $SO(10)$  unification [7], and we describe our assumptions on the unified framework in this section. When  $SO(10)$  is broken to  $SU(5)$ , we have small mixings among  $10$ ’s responsible for CKM mixing, while we need large mixings among  $\bar{5}$ ’s to explain the MNS mixing matrix [8] in the neutrino sector. Then back in the  $SO(10)$  multiplets, the top quark comes together with the near-maximal linear combination of second- and third-generation  $\bar{5}$ ’s. To the extent that we ignore all small Yukawa couplings except the top Yukawa coupling, the only effect of the Yukawa coupling appears in the multiplet

$$16_{i=3} \ni (t_L, V_{in} d_{Ln}) + (t_R)^c + U_{n3}^* (d_{Rn})^c + (\nu_{L3}, U_{n3}^* l_{Ln}) + V_{in} (l_{Rn})^c, \quad (4)$$

where we have use indices  $i, j, \cdot$  to represent the indices in  $Y_u^D$  basis while  $n, m, \cdot$  are reserved for the basis in which  $Y_d$  is diagonal. Here,  $V$  is the CKM matrix, and  $U$  the MNS matrix (some additional  $CP$  violating phases will be taken care of later), given the large  $U_{23}$  as evidenced in the atmospheric neutrino data, a near-maximal linear combination of  $s_R$  and  $b_R$  experiences the large top Yukawa coupling in the  $SO(10)$  theory. This simple point produces a large radiative correction to the soft mass of this linear combination, which is flavor-off-diagonal in the mass basis of down quarks. Therefore we can expect a potentially large effect in  $B_s$  mixing, and other related effects in  $B$  physics.

Our framework is closely related to that in [9] with the superpotential

$$W = \frac{1}{2} (Y_u)_{ij} 16_i 16_j 10_u + \frac{1}{2} (Y_d)_{ij} 16_i 16_j 10_d. \quad (5)$$

Here,  $Y_u, Y_d$  are up- and down-type Yukawa matrices,  $i, j = 1, 2, 3$  are generation indices, and  $10_u, 10_d$  are Higgs multiplets that contain  $H_u$  and  $H_d$  in the MSSM.

In addition to the above two terms,  $W$  must include some further (renormalizable or non renormalizable) Yukawa coupling responsible for the right-handed neutrino masses. As usual, this can be achieved either through  $\langle 126 \rangle$  or  $\langle \bar{16} \rangle^2 / M_{Pl}$ .

We need at least two Yukawa matrices  $Y_u$  and  $Y_d$  to generate intergenerational mixings and hence two Higgs multiplets. In this sense, this is the “minimal” framework of  $SO(10)$  unification. However, this makes both  $Y_u$  and  $Y_d$  matrices symmetric. A symmetric  $Y_u$  is acceptable phenomenologically, while a symmetric requirement for  $Y_d$  turns out to be too strict. The reason is simply that, in order to accommodate both CKM mixing among quarks and Maki-Nakagawa-Sakata (MNS) mixing among leptons, we need to set

$$Y_d = \Theta_L V_{CKM}^* Y_d^D \Theta_R U_{MNS} \Theta_\nu, \quad (6)$$

in the basis where  $Y_u$  and the right-handed neutrino mass matrix are diagonal (see below).  $Y_d^D = \text{diag}(Y_d, Y_s, Y_b)$  is the positive, diagonalized  $Y_d$  matrix.  $\Theta_L$ ,  $\Theta_R$  and  $\Theta_\nu$  are diagonal phase matrices. Because of this, we will instead take<sup>2</sup>

$$W = \frac{1}{2}(Y_u)_{ij}16_i16_j10_u + \frac{1}{2}(Y_d)_{ij}16_i16_j\frac{\langle 45 \rangle}{M_{Pl}}10_d. \quad (7)$$

Because of the combination of the Higgs multiplet 45, whose VEV  $\langle 45 \rangle \neq 0$  breaks  $SO(10)$ , and the Higgs in 10, the effective Yukawa coupling being either in 10 (symmetric between two 16's) or 120 (antisymmetric between two 16's) representations, the matrix  $Y_d$  can now have a mixed symmetry. We imagine that  $SO(10)$  is broken to  $SU(5)$  around  $10^{17}$  GeV, and this operator is large enough for the down-type Yukawa matrix. Note that we define  $V$  and  $U$  matrices to be in the CKM form with only one  $CP$  violating phase each. Phases in  $\Theta_L$  and  $\Theta_R$  are relevant only when the superheavy color triplet components of the Higgs multiplet is involved.  $\Theta_\nu$  is relevant for the  $CP$  violation in the neutrino sector when the Majorana character of the neutrino mass is involved.

Now we break  $SO(10)$  to  $SU(5)$ . Given a strong hierarchy among up-type Yukawa couplings and the large top Yukawa coupling, it is natural to stick to the basis where  $Y_u$  is diagonal,  $(Y_u)_{ij} = (Y_u^D)_i \delta_{ij}$ . Further decomposing multiplets under  $SU(5)$  as  $16_i = 10_i + \bar{5}_i + 1_i$ , and keeping only the Higgs multiplets  $5_u \in 10_u$  and  $\bar{5}_d \in 10_d$ , we find<sup>3</sup>

$$W = \frac{1}{2}(Y_u^D)_i 10_i 10_i 5_u + (Y_u^D)_i \bar{5}_i 1_i 5_u + (Y_d)_{ij} 10_i \bar{5}_j \bar{5}_d + \frac{1}{2} M_{ij} 1_i 1_j. \quad (8)$$

It is clear that, in the absence of the second term (neutrino Yukawa coupling), we can eliminate  $U_{MNS}$  entirely by changing the basis of  $\bar{5}_i$  in the  $SU(5)$  superpotential. This is an immediate way to see that the only effect of  $U_{MNS}$  is related to the neutrino mass. This, of course, is not necessarily true with the soft terms, which is the whole point of this paper.

Further breaking  $SU(5)$  down to the standard model, the prediction of this framework is that the Yukawa couplings in the MSSM+N (the MSSM together with right-handed neutrinos) are

$$W = (Y_u^D)_i Q_i U_i H_u + (Y_u^D)_i L_i N_i H_u + (V^* Y_d^D \Theta_R U \Theta_\nu)_{ij} Q_i D_j H_d + (V^* Y_d^D \Theta_R U \Theta_\nu)_{ij} E_i L_j H_d + \frac{1}{2} M_{ij} N_i N_j, \quad (9)$$

<sup>2</sup>The absence of renormalizable Yukawa coupling to  $10_d$  could well be a consequence of discrete symmetries.

<sup>3</sup>Right-handed neutrino masses arise from the coupling to  $SO(10)$ -breaking Higgs bosons either  $\langle 126 \rangle$  or  $\langle 16 \rangle^2 / M_{Pl}$ , as usual.

where we had absorbed the phases in  $\Theta_L$  into  $Q_i$  and  $E_i$ .  $Y_d$  is diagonalized by a biunitary rotation where the matrix acting on the left side represents the relative rotation of the left-handed down quarks with respect to the left-handed up quarks in the basis where the up quark mass matrix is diagonal. Hence, such matrix is just the usual CKM mixing matrix. The matrix acting on the right side represents the rotation to be performed on the left-handed leptons to go the physical basis of the charged leptons. It is easy to see that the phase matrix  $\Theta_\nu$  can be absorbed into the Majorana mass matrix  $M$  after redefining  $D_j$  and  $L_j$ , and, the phase matrix  $\Theta_R$  can be absorbed into  $(UD)$  multiplet or  $(EV^*)$  multiplet. The phase matrices  $\Theta_{L,R}$  are irrelevant as long as the colored triplet Higgs boson can be ignored as emphasized before. Hence such matrix  $U$  is to be identified with the neutrino mixing matrix  $U_{MNS}$  if we are in a basis where the physical light neutrinos are mass eigenstates. For this to happen, given that the neutrino Yukawa coupling matrix  $Y_u$  is diagonal, we have to assume that simultaneously also the right-handed neutrino mass matrix  $M$  is diagonal. Hence throughout our discussion we are taking  $Y_u$  and  $M$  simultaneously diagonal. Such a situation could result from simple  $U(1)$  family symmetries. As we will comment below, in an  $SO(10)$ -like scheme, with hierarchical  $Y_u^D$  and right-handed neutrino masses, the choice of having such simultaneous diagonalization looks rather plausible.<sup>4</sup>

The similarity of the charged lepton and down-quark Yukawa matrices is well-known phenomenologically. Quantitatively, the relation  $m_b = m_\tau$  could be indeed true at the unification scale, while  $m_s = m_\mu$ ,  $m_e = m_d$  are a factor of about three off. Here we take the point of view that the factors of three can be remedied by small  $SU(5)$ -breaking effects of the framework and do not worry about it. Clearly, lower-generation Yukawa couplings are subject to more corrections simply because their sizes are small. The  $B$ -physics signatures we will discuss do not depend on such details as we end up ignoring all Yukawa couplings except that of the top quark. It is important to notice that the order of left- and right-handed fields is the opposite between  $Q_i D_j$  and  $E_i L_j$  couplings.

The important outcome of this framework is the (approximate) equality of the neutrino and up-quark Yukawa matrices. The light neutrino masses, after integrating out the right-handed neutrinos in Eq. (9), are given by the superpotential

$$W = \frac{1}{2} (Y_u^D)_i (M^{-1})_{ij} (Y_u^D)_j (L_i H_u) (L_j H_u). \quad (10)$$

Since we assume that  $M$  is also diagonal in the same basis, this leads to the light Majorana neutrino mass matrix  $(m_\nu)_{nm} = (Y_u^D)_i^2 (v^2 / |M_i|) U_{ni}^* e^{-i\delta_i} U_{mi}^*$  in the basis where the charged lepton masses are diagonal, and  $e^{i\delta_i}$  is the phase of  $M_i$ . The immediate conclusion is that the right-handed neutrino Yukawa matrix must be roughly doubly hierarchical

<sup>4</sup>For a discussion of neutrino masses and mixings based on the simplest  $SO(10)$  mass relations and the seesaw mechanism, see the work of Ref. [10].



compared to the up-quark Yukawa matrix. Phenomenologically, the large angle Mikheyev-Smirnov-Wolfenstein (MSW) solution is the most promising solution to the solar neutrino problem. Then the two mass splittings [11]

$$\Delta m_{\oplus}^2 \simeq 3 \times 10^{-3} \text{ eV}^2, \quad (11)$$

$$\Delta m_{\odot}^2 \simeq 0.3 - 2 \times 10^{-4} \text{ eV}^2, \quad (12)$$

are not very different, especially after taking their square root. On the other hand, the up-quark Yukawa matrix has a strong hierarchy  $Y_u \ll Y_c \ll Y_t$ . To obtain similar mass eigenvalues between the largest and the 2nd largest eigenvalues as suggested by data, we need  $Y_c^2/M_2 \sim 0.2 Y_t^2/M_3$ . Moreover, the basis where the  $Y_u$  matrix is diagonal must be strongly correlated to the basis where the right-handed neutrino masses are diagonal to achieve this. The simplest possibility is to assume their simultaneous diagonalization, as we said above. Note also that all three physical  $CP$  violating phases associated with the light  $3 \times 3$  Majorana mass matrix are present in this model as free parameters.

At GUT scale, the top quark mass and the largest neutrino Dirac mass are equal. As a result the heaviest neutrino mass is  $m_t^2/M_3$ . From the recent fit to the Super-Kamiokande neutrino data and assuming nondegenerate neutrino masses, one has  $m_{\nu_3} \sim 0.05 \text{ eV}$ . For  $m_t \sim 178 \text{ GeV}$ , this corresponds to  $M_3$  of roughly  $10^{15} \text{ GeV}$  slightly below the GUT scale as expected. It is very interesting to see that the  $SO(10)$  model ties up neutrino mass, top mass and GUT scale nicely.

### B. Effects on soft masses

The size of the radiative corrections on the SUSY soft masses induced by the neutrino Yukawa couplings and their possible consequence on low-energy flavor physics had been studied within the  $SU(5)$  unification in Refs. [6,12–15]. Following these papers, we shall assume that above some GUT unification scale, the SUSY breaking parameters are universal and can be parameterized by the universal scalar mass  $m_0$ , the universal  $A$ -parameter  $a_0$  which is the ratio of the SUSY breaking trilinear scalar interaction to the corresponding Yukawa couplings, the  $B$  parameter entering the scalar bilinear term mixing the two Higgs doublets and the universal gaugino mass  $m_G$ . The scale,  $M_*$ , where these universal SUSY breaking values should be applied depends on the details of the SUSY breaking mechanism. Here we shall simply assume it to be near the Planck scale.

In the context of SUSY  $SU(5)$  [6,12], it was shown that, if the right-handed neutrino singlet is introduced to account for the data on neutrino oscillation, large neutrino Yukawa couplings involved in the neutrino Dirac masses, can induce large off-diagonal mixings in the right-handed down squark mass matrix through renormalization group evolution between  $M_*$  and  $M_{GUT}$ . In addition, the contributions to the scalar masses induced in the running by the neutrino Yukawa couplings will generally be complex with new  $CP$  violating phases unrelated to the Kobayashi-Maskawa (KM) phase in standard model. The above-mentioned mixings can be parametrized as  $\delta_{ij}^R = (m_{d_R}^2)_{ij}/m_{q^2}$  where  $m_{q^2}$  is the average

right-handed down squark mass. In particular, in Ref. [6], it was shown that the induced  $\delta_{12}^R$  is large enough to account for many of the observed  $CP$  violating phenomena in the kaon system providing an alternative to the CKM interpretation of these data. In Ref. [12], it was shown that  $\delta_{13}^R$  can give rise to a  $CP$  asymmetry in  $B_d \rightarrow \phi K_s$  much larger than the KM prediction.

Here we wish to point out first that the off-diagonal mixing parameter  $\delta_{23}^R$  is further enhanced in the context of a SUSY  $SO(10)$  model. In the next section we will elaborate on the phenomenological consequence of large  $\delta_{23}^R$ . In our case the constraint coming from the upper bound on  $BR(\mu \rightarrow e \gamma)$  turns out to be less severe than in the naive  $SU(5)$  context [6,12].

Because of the larger matter content of the  $SO(10)$  GUT model, the renormalization group evolution from  $M_*$  down to  $SO(10)$  breaking scale,  $M_{10}$ , is faster than that of the  $SU(5)$  model. The induced off-diagonal elements in the SUSY breaking mass matrix of the right-handed down squarks  $\tilde{d}_R$  are given by (in the basis in which  $Y_D$  is diagonal)

$$[m_{d_R}^2]_{nm} \simeq -\frac{1}{8\pi^2} [Y^{u\dagger} Y^u]_{nm} (3m_0^2 + a_0^2) \left( 5 \log \frac{M_*}{M_{10}} + \log \frac{M_{10}}{M_5} \right), \quad (13)$$

where  $M_5$  is the  $SU(5)$  breaking scale and

$$[Y^{u\dagger} Y^u]_{nm} = [\Theta_R U Y_u^{D2} U^\dagger \Theta_R^*]_{nm} = e^{-i(\phi_m^{(L)} - \phi_n^{(L)})} y_t^2 [U]_{m3}^* [U]_{n3}, \quad (14)$$

where  $e^{i\phi_n^{(L)}}$  is the phase from  $(\Theta_R)_{nn}$ . Note that these phases are not relevant to any other low energy physics.

To account for the large atmospheric neutrino mixing, the second and third entries of the third row of  $U_{MNS}$  should be of order  $1/\sqrt{2}$ , while the first entry,  $U_{e3}$ , is severely limited by the CHOOZ experiment:  $|U_{e3}| \leq 0.11$ . Hence we obtain

$$[Y^{u\dagger} Y^u]_{23} = 0.5 e^{-i(\phi_3^{(L)} - \phi_2^{(L)})} (m_{tG}/178 \text{ GeV})^2, \quad (15)$$

where  $m_{tG}$  is the top quark mass at  $M_G$ .

The factor 5 in the renormalization group (RG) coefficient above the  $SO(10)$  breaking scale is due to the contribution of the loop diagram with  $(10, \bar{5}_u)$  multiplets of  $SU(5)$  in the loop which is not present in  $SU(5)$ . They contribute four times more than the usual  $(1, \bar{5}_u)$  contribution in  $SU(5)$ , and  $\delta_{23}^R$  can easily be  $O(1)$ .

Note that  $m_{tG}$  can be quite different from the pole mass of about  $m_t \sim 178 \text{ GeV}$ . The evolution of  $m_{tG}$  between  $M_*$  and  $M_G$  has been discussed in the literature [16].

In  $SU(5)$  models, people had assumed that the right-handed neutrino mass matrix is given by an identity matrix to simplify the analysis. Given only a small hierarchy between  $\Delta m_{\oplus}^2$  and  $\Delta m_{\odot}^2$  for large mixing angle MSW solution,

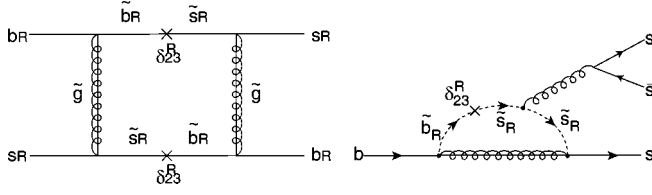


FIG. 1. Possible important contributions to  $B$  physics from large  $\tilde{b}_R$ - $\tilde{s}_R$  mixing, such as  $B_s$ - $\bar{B}_s$  mixing, and SUSY penguin contributions to  $B_d \rightarrow \phi K_s$  transition.

the second-generation neutrino Yukawa coupling is sizable, with a large mixing with the first-generation states. This led to quite stringent constraints from the processes such as  $\mu \rightarrow e \gamma$ . In our  $SO(10)$  framework, however, the neutrino Yukawa matrix is as hierarchical as the up-quark sector, and only the third-generation Yukawa coupling is significant. In the third-generation multiplet, the electron state appears with a suppressed coefficient  $U_{e3}$ . Therefore, unlike the frameworks studied in the literature, contributions to processes that involve the first generation (like  $\mu \rightarrow e + \gamma$ ) are suppressed by this unknown element in the MNS matrix. In view of this fact, we prefer to focus on flavor violating processes involving the mixing between second- and third-generation in this paper.

One comment before we proceed to  $B$  physics. For the  $\tau \rightarrow \mu + \gamma$  process, the result of a recent comprehensive analysis of lepton flavor changing processes in Ref. [15] is applicable to our case. It was found that the branching ratio can be as large as  $10^{-8}$ , when the Yukawa coupling is  $O(h_t)$  as suggested by  $SO(10)$ . It is interesting to note that since  $\mu \rightarrow e + \gamma$  is suppressed due to hierarchical Yukawa coupling of the first two generations and small  $U_{e3}$ , there is a wide range of parameters such as to give  $Br(\tau \rightarrow \mu + \gamma)$  observable at  $B$ -factory experiments consistent with the  $\mu \rightarrow e + \gamma$  constraint [17].

#### IV. CONSEQUENCES IN $B$ PHYSICS

In this section we present some implications of a large and complex  $\delta_{23}^R$  in  $B$  physics (see Fig. 1).

The diagrammatic contributions of  $\delta_{23}^R$  to various  $\Delta B = -\Delta S = 2$  and  $\Delta B = -\Delta S = 1$  processes were worked out in detail in Ref. [18].<sup>5</sup> In particular a complex  $\delta_{23}^R$  can play a major role in  $CP$  violating  $B$  decays [20,12,21,14,22].

The first effect of a conspicuous  $\delta_{23}^R$  would be a large contribution to the  $\Delta B = -\Delta S = 2$ ,  $B_s$ - $\bar{B}_s$  mixing through the operator  $Q_1 = \bar{s}_R^\alpha \gamma_\mu b_R^\alpha \bar{s}_R^\beta \gamma^\mu b_R^\beta$  with complex coefficient

$$\mathcal{H}_{eff} = -\frac{\alpha_s^2}{216m_q^2} (\delta_{23}^R)^2 (24Q_1 x f_6(x) + 66Q_1 \tilde{f}_6(x)), \quad (16)$$

<sup>5</sup>For an updated analysis of the gluino-mediated SUSY contributions to the  $B_d$ - $\bar{B}_d$  mixing and to the  $CP$  asymmetry in the decay  $B \rightarrow J/\psi K_s$  including the NLO QCD corrections and  $B$  coefficients as computed in the lattice instead of using the vacuum insertion approximation, see Ref. [19].

where the functions  $f_6$  and  $\tilde{f}_6$  are defined as in Ref. [18].

From Eqs. (13),(15) we see that  $\delta_{23}^R$  can easily be as large as 0.5, yielding a SUSY contribution to  $\Delta M_s$  comparable to that of the SM. Hence, in our scheme the operator gives rise to a large  $B_s$ - $\bar{B}_s$  mixing with a complex phase which is almost unconstrained so far. In contrast, the SM contribution to  $B_s$ - $\bar{B}_s$  mixing has very small phase (in the usual Wolfenstein convention). This gives rise to many phenomenological consequences as will be sampled below.

Secondly,  $\delta_{23}^R$  gives rise to new contribution to direct  $B$  decays. Two categories of contributions are more important. The first one is a  $\Delta B = 1$  box diagram contribution. The second is of the type of electromagnetic or gluonic penguin contributions. There are also contribution of electroweak penguin type which we shall ignore because they are generally smaller.

In  $B$  decay processes in which the dominant contribution in SM is at the tree level, the additional contribution due to  $\delta_{23}^R$  can at most be a small percentage. This is true even if the SM contribution comes with strong mixing angle suppression [22] such as  $B^\pm \rightarrow DK^\pm$  or  $\bar{D}K^\pm$ . However, if the initial state mixing (such as  $B_s$  mixing) plays a strong role in the phenomena, then  $\delta_{23}^R$  can significantly alter the phenomenology.

In  $B$  decay processes in which the dominant contributions involve one loop contributions, such as the penguin diagrams, one should expect large additional contribution due to the  $\delta_{23}^R$  in both amplitude and  $CP$  asymmetry.

As an application of the above analysis, we can roughly classify the phenomenology into three categories.

(1) Measurements of the  $\beta$  angle of the unitarity triangle. The leading mode,  $B \rightarrow J/\psi K_s$ , has large phase from the initial state  $B_d$  mixing (in Wolfenstein convention), and large real tree level decay. It therefore does not receive significant contribution from  $\delta_{23}^R$ . However, other modes, such as  $B_d \rightarrow \phi K_s$  ( $\bar{b} \rightarrow \bar{s} c \bar{c}$ ), which, in SM, measures the same  $\beta$ , can now receive large additional contribution from  $\delta_{23}^R$  through the penguin diagrams [12,21]. The fractional phase difference  $r_\beta = (\beta(JK_s) - \beta(\phi K_s)) / \beta(JK_s)$  is a measure of  $\delta_{23}^R$  that can be as large as 50%.

(2) Measurements of the  $\gamma$  angle of the unitarity triangle. One class of popular measurements on  $\gamma$  involves  $B_s$  decays [23]. In the SM, the  $B_s$  mixing is real to a good approximation. Therefore any measurements of  $\gamma$  using  $B_s$  decays will be strongly affected by  $\delta_{23}^R$ . For example, in  $\bar{B}_s \rightarrow (D_s)^- K^+$  the  $CP$  asymmetry is due to the interference of the  $B_s$  decays, which has real amplitude in SM, and the  $\bar{B}_s$  decay with complex amplitude after the  $B_s$ - $\bar{B}_s$  mixing. The two decays are roughly of equal magnitude and the phase of  $\bar{B}_s$  is exactly  $\gamma$  in SM. With  $\delta_{23}^R$ , even the  $\bar{B}_s \rightarrow B_s \rightarrow (D_s)^- K^+$  develops a large phase due to additional complex contribution to the mixing. The phase is proportional to  $\arg(M_{12}(\delta_{23}^R) / [M_{12}(\delta_{23}^R) + M_{12}(\text{SM})])$  where  $M_{12}(\text{SM})$  and  $M_{12}(\delta_{23}^R)$  are the  $B_s$  mixing amplitude of the SM and that due to  $\delta_{23}^R$  respectively [22].

Another class of measurement on  $\gamma$  are using charge  $B$  decays. For example in  $B^+ \rightarrow D^0 K^+$  or  $\bar{D}^0 K^+$ ,  $\gamma$  is measured through the interference between the two quark level processes  $b \rightarrow c \bar{u} s$  and  $b \rightarrow u \bar{c} s$ . Along the decay chain the  $c$  or  $\bar{c}$  produce in the final state a  $D^0$  or  $\bar{D}^0$  mesons respectively. The two contributions interfere if both  $D^0$  or  $\bar{D}^0$  decay to the same final state  $f_D$  and have a relative phase  $\gamma$ . Here,  $f_D$  is one of the states that both  $D$  and  $\bar{D}$  can decay into, such as  $K^- \pi^+$  or  $CP$  eigenstates  $K^+ K^-$ ,  $\pi^+ \pi^-$ ,  $K_s \pi^0$  or  $K_s \phi$  [22]. In this measurement, the role played by  $\delta_{23}^R$  is negligible, so it measures the same value as in SM. Therefore, just like  $\beta$  measurements, by comparing  $\gamma$  measurements in  $B_s$  and in  $B^\pm$  decays, one can get a measure of  $\delta_{23}^R$ . In principle, by comparing  $r_\beta$  with  $r_\gamma$ , which is similarly defined, one can get a strong evidence of the existence of large  $\delta_{23}^R$ .

(3) Decays which are expected to be essentially  $CP$  conserving in the SM. Some of decays may have large  $CP$  asymmetry due to the existence of  $\delta_{23}^R$ . Examples:  $B_s \rightarrow J \phi$  or  $B_s \rightarrow (D_s)^+ (D_s)^-$  or  $B \rightarrow X_s \gamma$ .

## V. CONCLUSION

In this paper, we pointed out that a large mixing between  $\nu_\tau$  and  $\nu_\mu$  as observed in atmospheric neutrino oscillation

may lead to a large mixing between  $\tilde{b}_R$  and  $\tilde{s}_R$  because they belong to the same  $SU(5)$  multiplets. This occurs naturally in  $SO(10)$  grand unified models which we have described in detail. These models do not give rise to dangerously large  $\mu \rightarrow e \gamma$  and similar processes which involve the first generation, given the current limit on  $U_{e3}$  from reactor-neutrino experiments. A large mixing between  $\tilde{b}_R$  and  $\tilde{s}_R$  leads to interesting effects in  $B$  physics, such as large and  $CP$ -violating  $B_s$  mixing, different “ $\sin 2\beta$ ” between  $B_d \rightarrow \phi K_s$  and  $J/\psi K_s$ , different “ $\gamma$ ” from various measurements, and  $CP$  asymmetry in  $B_s \rightarrow J \phi$ ,  $(D_s)^+ (D_s)^-$ .

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